5525: **Proposed by Daniel Sitaru**, **National Economic College "Theodor Costescu"**, **Drobeta Turnu-Severin**, **Mehedinti**, **Romania**

Find real values for x and y such that:

$$4\sin^2(x+y) = 1+4\cos^2 x+4\cos^2 y.$$

Solution by Arkady Alt, **San Jose**,**California**, **USA**.

Since $2\cos^2 x+2\cos^2 y = 2+\cos 2x+\cos 2y = 2+2\cos(x+y)\cos(x-y)$

then $4\sin^2(x+y) = 1+4\cos^2 x+4\cos^2 y \Leftrightarrow$

$$4\sin^2(x+y) = 5+4\cos^2 x+4\cos(x+y)\cos(x-y) \Leftrightarrow$$

$$1+4\cos(x+y)\cos(x-y)+4\cos^2(x+y) = 0 \Leftrightarrow$$

$$\sin^2(x-y)+(2\cos(x+y)+\cos(x-y))^2 = 0 \Leftrightarrow$$

$$\begin{cases} \sin(x-y) = 0 \\ 2\cos(x+y)+\cos(x-y) = 0 \end{cases} . \Leftrightarrow \begin{cases} x-y = n\pi, n \in \mathbb{Z} \\ \cos(x+y) = \dfrac{(-1)^{n+1}}{2} \end{cases} \Leftrightarrow$$

$$\begin{cases} x-y = n\pi, n \in \mathbb{Z} \\ x+y = \pm\dfrac{(3+(-1)^n)\pi}{6}+2k\pi, k \in \mathbb{Z} \end{cases} \Leftrightarrow$$

$$(x,y) = \left(\pm\frac{(3+(-1)^n)\pi}{12}+k\pi+\frac{n\pi}{2}, \pm\frac{(3+(-1)^n)\pi}{12}+k\pi-\frac{n\pi}{2}\right), n,k \in \mathbb{Z}.$$